Certain Mean Values in the Theory of the Traveling-Wave Amplifier*

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Some simple results relating to certain mean values are given. It is not assumed that the signals are necessarily small; hence nonlinear effects are taken into account.

The purpose of this note is to give a few simple results relating to certain mean values that occur in the theory of the traveling-wave amplifier. Whereas all of the previous theory of the amplifier has been based on the assumption that the signals are small, so that the system behaves effectively as a linear system, no such assumption is involved in the results given here.

After some idealization of the physical system, the fundamental equations of the traveling-wave amplifer can be written as follows:[2]

$$L\frac{\partial I}{\partial t} + RI = -\frac{\partial V}{\partial x}, \tag{1}$$

$$C\frac{\partial V}{\partial t} = -\frac{\partial I}{\partial x} + \alpha\frac{\partial \rho}{\partial t}, \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0, \tag{3}$$

$$\frac{\partial v}{\partial t} + \frac{1}{2}\frac{\partial v^2}{\partial x} = \beta\frac{\partial V}{\partial x}. \tag{4}$$

The independent variables t and x represent, respectively, time and distance measured in the axial direction from the driving point; I, V, ρ and v denote the instantaneous local values of the current in the conductor, the potential of the conductor, the linear charge density of the

* This material was prepared as a memorandum or report in 1946, but was never published. It has, however, been known to people working in the field, and it has been mentioned in published work.[1] It seems desirable that it be made generally available in its original form.

electron stream, and the velocity of the electrons, respectively; L, R, C, α and β are constants.

Now suppose that we have a state of the system in which, for each value of x, the variables I, V, ρ and v are all periodic functions of time, with the period T. (It is to be noted that nothing is assumed about the waveforms of these periodic functions.)

By (1), we have the relation

$$L \frac{1}{T} \int_{t_0}^{t_0+T} \frac{\partial I}{\partial t} dt + R \frac{1}{T} \int_{t_0}^{t_0+T} I \, dt = - \frac{\partial}{\partial x} \frac{1}{T} \int_{t_0}^{t_0+T} V dt, \qquad (5)$$

where t_0 is an arbitrary constant.

The first term in the left-hand member of (5) vanishes, because

$$\int_{t_0}^{t_0+T} \frac{\partial I}{\partial t} dt = I(x, t_0 + T) - I(x, t_0),$$

and because I is periodic with respect to t with the period T. The expressions

$$\frac{1}{T} \int_{t_0}^{t_0+T} I \, dt \qquad \text{and} \qquad \frac{1}{T} \int_{t_0}^{t_0+T} V \, dt,$$

which we shall denote by the symbols $\bar{I}$ and $\bar{V}$ respectively, are the means of I and V with respect to t over the period T, for an arbitrary value of x. A bar over a letter is used in this sense throughout the discussion.

Thus we have the relation

$$R\bar{I} = - \frac{d\bar{V}}{dx}. \qquad (6)$$

Similarly, from (2), (3) and (4), we get the relations

$$\frac{d\bar{I}}{dx} = 0, \qquad (7)$$

$$\frac{d(\overline{\rho v})}{dx} = 0, \qquad (8)$$

$$\frac{d\overline{v^2}}{dx} = 2\beta \frac{d\bar{V}}{dx}. \qquad (9)$$

The general solution of (6), (7), (8) and (9) is

$$\bar{I} = K_1,$$

$$\bar{V} = -K_1 R x + K_2,$$

$$\overline{\rho v} = K_3 ,$$
$$\overline{v^2} = -2K_1\beta Rx + 2K_2\beta + K_4 ,$$

where the K's are arbitrary constants.

The most interesting and important state of the system is that in which, at the driving end, we have

$$\bar{I} = 0, \qquad \bar{V} = 0, \qquad \rho = \rho_0 , \qquad v = v_0 , \tag{10}$$

where ρ_0 and v_0 are constants. In this state, and for any value of x, we have the relations

$$\bar{I} = 0, \qquad \bar{V} = 0, \qquad \overline{\rho v} = \rho_0 v_0 , \qquad \overline{v^2} = v_0^2 . \tag{11}$$

This result can be stated in words as follows: If at the driving point the mean values of the conductor current and voltage are zero, and if at the same point ρ and v have the constant values ρ_0 and v_0 , then the mean values of the conductor current and voltage are everywhere zero, the mean value of the electron convection current is everywhere $\rho_0 v_0$, and the mean value of the square of the electron velocity is everywhere v_0^2.

We note that, although the system is nonlinear, there is no rectification of the applied signals.

By the Schwarz inequality, we have the relation

$$(\overline{\rho v})^2 \leqq (\overline{\rho^2})(\overline{v^2}).$$

This, together with the relations $\overline{\rho v} = \rho_0 v_0$ and $\overline{v^2} = v_0^2$, implies that, in the state to which the equations (10) relate, we have everywhere the relation

$$\overline{\rho^2} \geqq \rho_0^2. \tag{12}$$

By the Schwarz inequality, we also have the relation

$$(\overline{1 \cdot v})^2 \leqq (\overline{1^2})(\overline{v^2}) = \overline{v^2};$$

and this, together with the relation $\overline{v^2} = v_0^2$, gives us the relation

$$|\bar{v}| \leqq v_0 . \tag{13}$$

REFERENCES

1. Nordsieck, A., Theory of Large-Signal Behavior of Traveling-Wave Amplifiers, Proc. I.R.E., **41**, 1953, p. 630.
2. Pierce, J. R., *Traveling-Wave Tubes*, D. Van Nostrand Co., New York, 1950.